Instituto Português da Qualidade

LABORATÓRIO NACIONAL DE ENGENHARIA CIVIL

CALCULATION OF THE FLOW-RATE MEASUREMENT UNCERTAINTY BY MEANS OF PITOT TUBES USING THE MONTE CARLO METHOD

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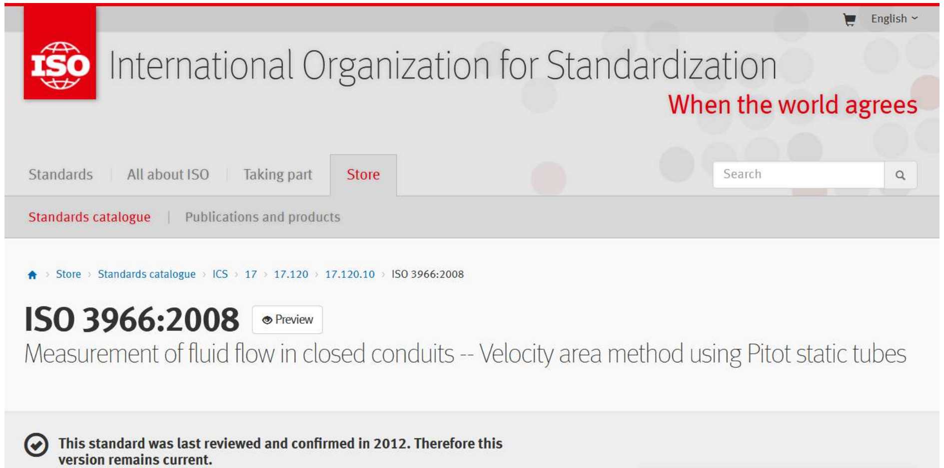
Summary

- Introduction
- Velocity and flow-rate measurements by means of Pitot tubes
- Measurement uncertainty evaluation
  - Input probabilistic formulation and uncertainty propagation stages
  - Monte Carlo Method
- Results
  - Air density
  - Compressibility correction factor
  - Local velocity
  - Volumetric and mass flow-rate
- Conclusions

Introduction

- Measurement of fluid flow in closed conduits
  - Point velocity methods
    - Hot-wire and hot-film anemometers
    - Vane anemometers
    - Current meters
    - Laser velocimetry
    - **Pitot tubes**
  - Normative framework: **ISO 3966:2008**
    - Design and maintenance
    - Calculation procedures
    - Annex G – Example of estimation of the uncertainty
      - Conventional error approach
  - Motivation: review and update the procedure used to evaluate the measurement accuracy in flow-rate measurements using Pitot tubes



2008 VIM and GUM Supplement 1 editions

Introduction

- Application of the Monte Carlo Method
  - Non-linearity and complexity of the applied mathematical models
  - Based on the input information shown in ISO 3966 Annex G
    - Robust comparison with the results obtained using the conventional error approach
    - The quantified flow-rate measurement accuracy is merely illustrative: in a real case scenario, the presented probabilistic formulation and quantification must be confirmed and updated if required

Velocity and flow-rate measurements by means of Pitot tubes

- Bernoulli's principle
  - Local fluid velocity, *v*

$$v = \alpha \cdot \sqrt{\frac{2 \cdot \Delta p}{\rho}}$$

- Calibration factor, *α*
- Differential pressure, Δ*p*
- Compressible fluid density, *ρ*

$$\rho = \frac{p \cdot M}{Z \cdot R_g \cdot T}$$

- Local static pressure, *p*
- Molar mass of the fluid, *M*
- Molar constant of gas, *Rg*
- Gas law deviation, *Z*
- Local static temperature, *T*

- For air velocities above 60 m·s^{-1}

$$v = \alpha \cdot (1 - \varepsilon) \cdot \sqrt{\frac{2 \cdot \Delta p}{\rho}}$$

- Compressibility correction factor, (1 - *ε*)

$$(1 - \varepsilon) \approx \left[1 - \frac{1}{2 \cdot \gamma} \cdot \frac{\Delta p}{p} + \frac{\gamma - 1}{6 \cdot \gamma^2} \left(\frac{\Delta p}{p}\right)^2\right]^{\frac{1}{2}}$$

- Ratio of specific heat capacities, *γ*, (varies between 1,1 and 1,7)

- Volumetric flow-rate

$$q_V = U \cdot A$$

- Discharge velocity, *U*

ISO 3966:2008

- Graphical integration
- Numerical integration
- Arithmetical methods

$$U = \frac{1}{\pi \cdot R^2} \int_0^{2\pi} \int_0^{R} v(r, \theta) \cdot r \cdot \mathrm{d}r \cdot \mathrm{d}\theta$$

$$U = \int_0^1 \int_0^1 v \cdot \mathrm{d}\left(\frac{h}{H}\right) \cdot \mathrm{d}\left(\frac{l}{L}\right)$$

- Cross-section, *A*

- Mass flow-rate

$$q_m = q_V \cdot \rho$$

Measurement uncertainty evaluation

- Input probabilistic formulation and uncertainty propagation stages

  - Uncertainty propagation stages
    I. Fluid density
    II. Local velocity
    III. Volumetric flow-rate
    IV. Mass flow-rate

Estimates related to input quantities

p	105 000 Pa	γ	1,40
M	28 963,5 ·10^{-6} kg·mol^{-1}	Δp	10,00 Pa
Z	1,000 00	$\xi*$	0,00 Pa
R_g	8,314 459 8 J·mol^{-1}·K^{-1}	α	1,000
T	290,0 K	A	0,120 0 m^2

* Correction of the measured differential pressure related to the head loss.

Uncertainty components related to input quantities

Uncertainty component	Uncertainty source	PDF	Standard uncertainty
$u(p)$	Local static pressure	Gaussian	100 Pa
$u(M)$	Molar mass of the fluid	Gaussian	2,3·10^{-6} kg·mol^{-1}
$u(Z)$	Gas deviation factor	Uniform	0,000 5/√3
$u(R_g)$	Molar constant of gas	Gaussian	4,8·10^{-6} J·mol^{-1}·K^{-1}
$u(T)$	Local static temperature	Gaussian	0,1 K
$u(\Delta p)_{inst}$	Instrumental uncertainty	Gaussian	0,4%
$u(\Delta p)_{head}$	Correction for head loss	Gaussian	0,2%
$u(\alpha)$	Calibration factor of the Pitot tube	Gaussian	0,2%
$u(\gamma)$	Ratio of specific heat capacities	Uniform	0,3/√3
$u(A)$	Cross-section area	Gaussian	0,2%

Measurement uncertainty evaluation

- Input probabilistic formulation and uncertainty propoagation stages

Additional uncertainty components related to the local velocity measurement

Uncertainty component	Uncertainty source	PDF	Standard uncertainty
$u(v)_{\text{block}}$	Blockage effect	Gaussian	0,25%
$u(v)_{\text{high}}$	Turbulence and high frequency fluctuations	Gaussian	0,50%
$u(v)_{\text{incl}}$	Pitot tube inclination	Gaussian	0,15%
$u(v)_{\text{grad}}$	Gradient velocity	Gaussian	0,15%
$u(v)_{\text{slow}}$	Slow fluctuations	Gaussian	0,10%

Additional uncertainty components related to the volumetric flow-rate measurement

Uncertainty component	Uncertainty source	PDF	Standard uncertainty
$u(q_V)_{\text{integ}}$	Integration technique	Gaussian	0,10%
$u(q_V)_{\text{rough}}$	Roughness coefficient estimate	Gaussian	0,05%
$u(q_V)_{\text{point}}$	Insufficient number of points	Gaussian	0,10%
$u(q_V)_{\text{posit}}$	Pitot tube positioning	Gaussian	0,05%

Measurement uncertainty evaluation

- Monte Carlo Method (MCM)
  - Computational simulation algorithm developed in Matlab®
  - Supplement 1 of the GUM
  - Mersenne-Twister pseudo-random number generator
  - Validated computational routines
    - Average
    - Mode
    - 95% expanded measurement uncertainties (absolute and relative)
    - Computational accuracy
    - Normalized histograms of the simulated output PDFs
  - Number of trials for convergent solutions: 10^6

Results

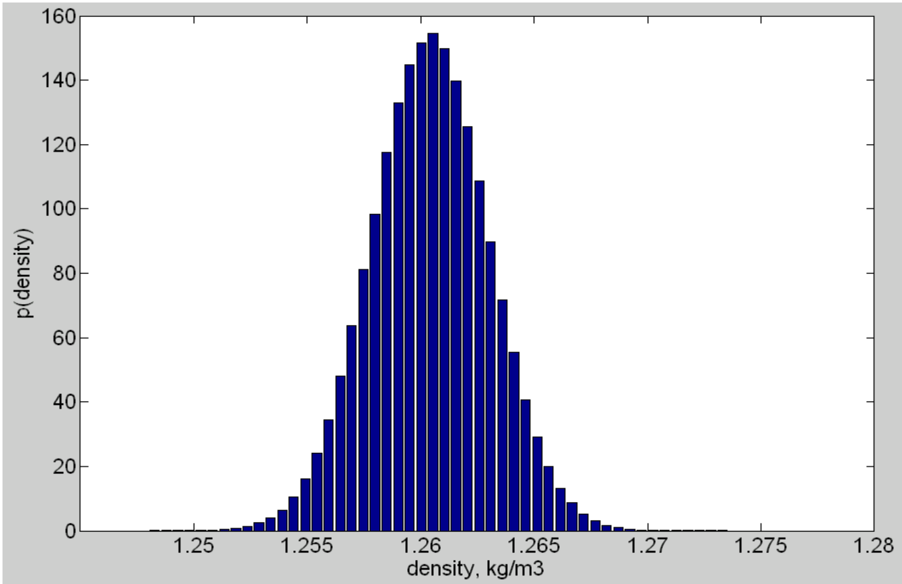
- Air density

  - Main contributions for the obtained uncertainty
    - Local static pressure (89%)
    - Local temperature (8%)
    - Molar constant of gas (3%)
    - Molar mass and gas deviation factor (negligible)

  - Local static pressure (800 mbar – 1200 mbar)
    - Air densities from (0,960 0 ± 0,004 9) kg·m^{-3} up to (1,440 6 ± 0,005 2) kg·m^{-3}

  - Local temperature (273 K – 373 K)
    - Constant air density 95% expanded uncertainty: 0,4%

Mean / kg·m^{-3}	Mode / kg·m^{-3}	$U_{95\%}$ / kg·m^{-3}	$U_{r\ 95\%}$ / %	$U_{c\ 95\%}$ / kg·m^{-3}
1,260 4	1,260 5	0,005 1	0,4	< 0,000 03

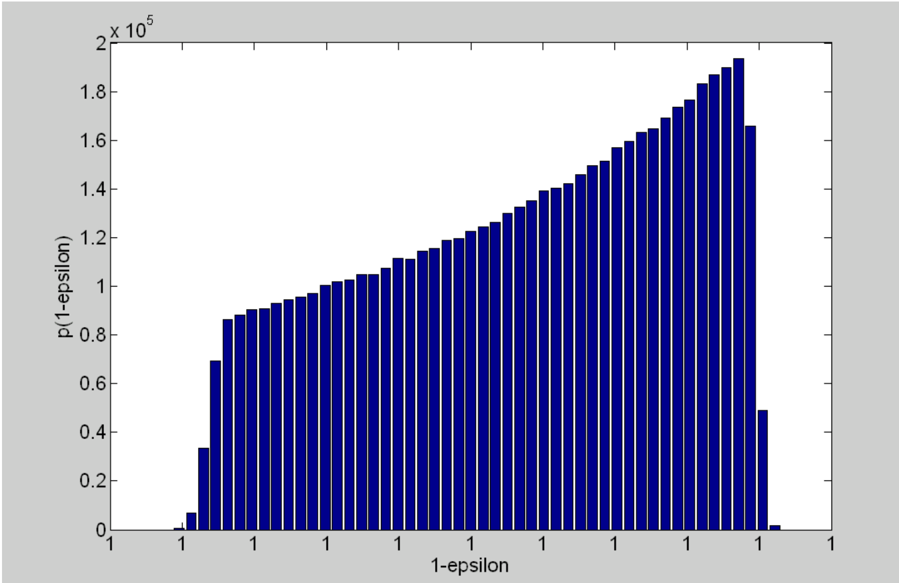


Results

- Compressibility correction factor

Mean	Mode	$U_{95\%}$	$U_{c\ 95\%}$
0,999 982 7	0,999 985 7	$3{,}6 \cdot 10^{-6}$	$< 7 \cdot 10^{-9}$

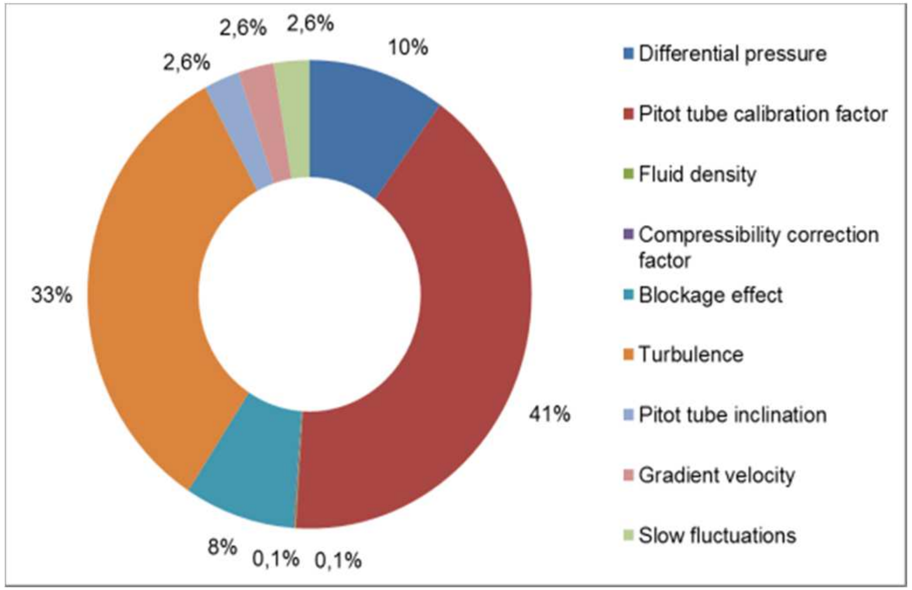
  - Main contributions for the obtained uncertainty
    - Ratio of specific heat capacities (99,7%)
    - Differential pressure (0,15%)
    - Local static pressure (0,15%)
  - Asymmetrical output PDF
    - Related to the ratio of specific heat capacities (a Gaussian shape PDF is obtained when considering this uncertainty component null)

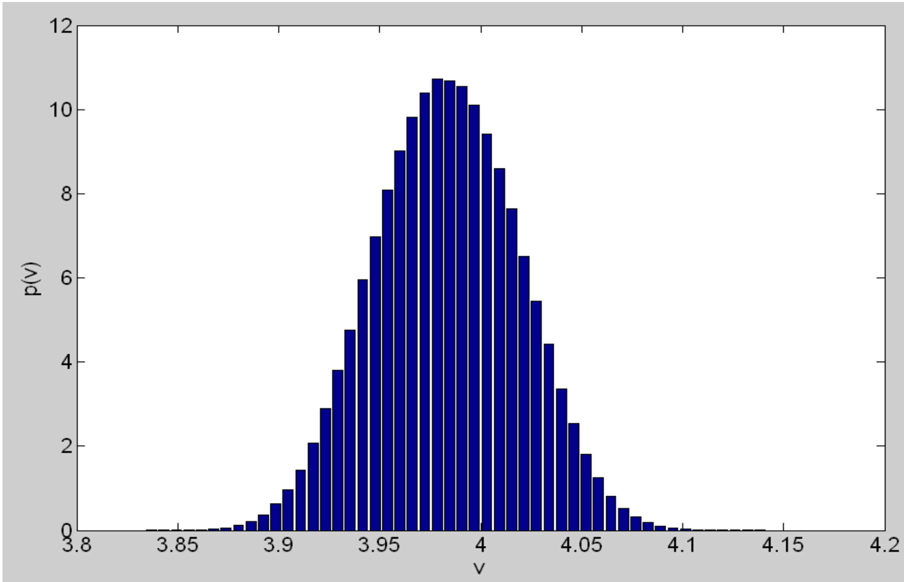


Results

- Local velocity
  - Studied measurement interval: 4 m·s^{-1} up to 90 m·s^{-1}
  - Constant relative 95% expanded uncertainty: 1,7%

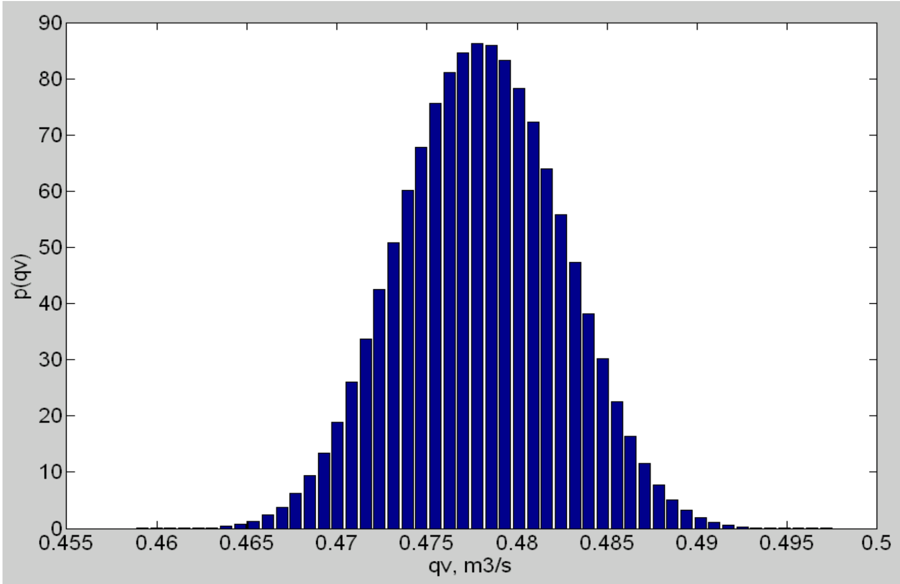
Mean / m·s^{-1}	Mode / m·s^{-1}	$U_{95\%}$ / m·s^{-1}	$U_{r\ 95\%}$ / %	$U_{c\ 95\%}$ / m·s^{-1}
3,983	3,978	0,069	1,7	< 0,000 4





Results

- Volumetric flow-rate
  - Main contributions for the obtained uncertainty
    - Local velocity (91%)
    - Cross-section area (4%)
    - Uncertainty components
      - Integration technique (1,5%)
      - Pitot tube positioning (1,5%)
      - Number of measurement points (1,5%)
      - Estimation of roughness coefficient (0,5%)
  - Studied measurement interval: 0,15 $m^3 \cdot s^{-1}$ (544 $m^3 \cdot h^{-1}$) up to 1,5 $m^3 \cdot s^{-1}$ (5440 $m^3 \cdot h^{-1}$)
  - Constant relative 95% expanded uncertainty: 1,8%



Mean / $m^3 \cdot s^{-1}$	Mode / $m^3 \cdot s^{-1}$	$U_{95\%}$ / $m^3 \cdot s^{-1}$	$U_{r\,95\%}$ / %	$U_{c\,95\%}$ / $m^3 \cdot s^{-1}$
0,478 0	0,477 9	0,008 7	1,8	< 0,000 05
Mean / $m^3 \cdot h^{-1}$	**Mode / $m^3 \cdot h^{-1}$**	**$U_{95\%}$ / $m^3 \cdot h^{-1}$**	**$U_{r\,95\%}$ / %**	**$U_{c\,95\%}$ / $m^3 \cdot h^{-1}$**
1721	1720	31	1,8	< 0,18

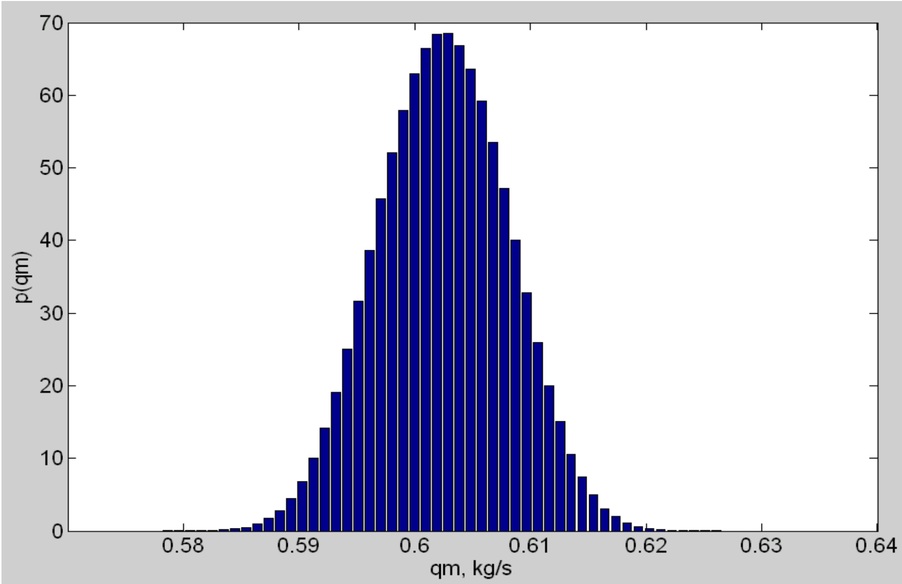
Results

- Mass flow-rate

  - Main contributions for the obtained uncertainty
    - Volumetric flow-rate (99%)
    - Fluid density (1%)

  - Studied measurement interval: 0,2 $kg \cdot s^{-1}$ (720 $kg \cdot h^{-1}$) up to 2,0 $kg \cdot s^{-1}$ (7200 $kg \cdot h^{-1}$)

  - Constant relative 95% expanded uncertainty: 1,8%



Mean / $kg \cdot s^{-1}$	**Mode** / $kg \cdot s^{-1}$	$U_{95\%}$ / $kg \cdot s^{-1}$	$U_{r\ 95\%}$ / %	$U_{c\ 95\%}$ / $kg \cdot s^{-1}$
0,602	0,603	0,011	1,8	< 0,000 06
Mean / $kg \cdot h^{-1}$	**Mode** / $kg \cdot h^{-1}$	$U_{95\%}$ / $kg \cdot h^{-1}$	$U_{r\ 95\%}$ / %	$U_{c\ 95\%}$ / $kg \cdot h^{-1}$
2167	2170	40	1,8	< 0,22

Conclusions

- Monte Carlo Method – fit-for-purpose
- 95% expanded relative measurement uncertainties
  - 1,7% - local velocity (1,4% in ISO 3966:2008)
  - 1,8% - volumetric and mass flow-rates (1,5% in ISO 3966:2008)
- Weak non-linearity of the applied mathematical models
- Low contribution of measurement uncertainty of quantities such as the compressibility correction factor
- Major contributions for the flow-rate measurement accuracy
  - Pitot tube calibration factor
  - Uncertainty component related to turbulence and high frequency fluctuations
- Future revision of ISO 3966:2008 standard – Annex G

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Thank you for your attention!

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